
Government Polytechnic
Educational Society, Gurugram

Functions and their Limits

Types of Functions

- Algebraic Functions : $f(x) = x^2+6x=2$
- Trigonometric Functions : $f(x) = \text{Sin } x$
- Inverse Trigonometric Functions : $f(x) = \text{Sin}^{-1}x$
- Exponential Functions : $f(x) = a^x$
- Logarithmic Functions : $f(x) = \log_a x$

LIMITS

The Algebra of limits

- If f(x)= k, a constant function then $\underset{x \to a}{Lt} f(x) = k$

METHORDS OF FINDING LIMITS

Direct Substitution

Example Limits

Use the rules from the previous slides to find the following limits using direct substitution.

a) $\lim_{x \to 5} (4x^2 - 2x + 6)$

$$\lim_{x \to 5} \left(4x^2 - 2x + 6\right) = 4(5)^2 - 2(5) + 6 = 4(25) - 10 + 6 = 100 - 10 + 6 = 96$$

b) $\lim_{x \to 1} \frac{x^2 + x + 2}{x + 1}$

$$\lim_{x \to 1} \frac{x^2 + x + 2}{x + 1} = \frac{1^2 + 1 + 2}{1 + 1} = \frac{4}{2} = 2$$

FACTORIZATION METHORD

WHEN WE SUBSTITUTE THE VALUE OF X IN THE RATIONAL EXPRESSION IT TAKES THE FORM $\frac{0}{0}$.

For example: $\lim_{x\to 3} \frac{x^2-x-6}{x^3-3x^2+x-3}$ $\left[\frac{0}{0} \text{ form}\right]$

$$=\lim_{x\to 3} \frac{(x-3)(x+2)}{x^2(x-3)+1(x-3)}$$

$$=\lim_{x\to 3} \frac{(x-3)(x+2)}{(x^2+1)(x-3)}$$

$$=\lim_{x\to 3} \frac{x-2}{x^2+1}=\frac{3-2}{3^2+1}=\frac{1}{10}$$

RATIONALIZATION METHORD

$$\lim_{x\to 7}\frac{\sqrt{x+2}-3}{x-7}=\lim_{x\to 7}\frac{\left(\sqrt{x+2}-3\right)}{(x-7)}\cdot\frac{\left(\sqrt{x+2}+3\right)}{\left(\sqrt{x+2}+3\right)}$$

$$=\lim_{x\to 7}\frac{(x+2-9)}{(x-7)\cdot\left(\sqrt{x+2}+3\right)}$$

$$=\lim_{x\to 7}\frac{(x-7)}{(x-7)\cdot\left(\sqrt{x+2}+3\right)}$$

$$=\lim_{x\to 7}\frac{1}{\left(\sqrt{x+2}+3\right)}$$

$$=\frac{1}{\sqrt{7+2}+3}$$

$$=\frac{1}{6}$$

EVALUATION OF TRIGNOMETRIC LIMITS

$$\lim_{x \to 0} \frac{\sin 4x}{x} = \lim_{x \to 0} \frac{4 \sin 4x}{4x}$$

$$= \left(\lim_{x \to 0} 4 \right) \cdot \lim_{x \to 0} \frac{\sin 4x}{4x}$$

$$= 4 \cdot 1$$

$$\left(\lim_{x \to 0} \frac{\sin x}{x} = 1 \right)$$

$$\lim_{x \to 0} \frac{\sin 4x}{x} = 4$$

LIMITS AT INFINITY

$$\underset{x\to\infty}{\text{Lt}} \frac{x^7+x^3+1}{x^8-3x^6+5}$$

Sol. $\underset{x\to\infty}{\text{Lt}} \dfrac{x^7+x^3+1}{x^8-3x^6+5} = \dfrac{(\infty)^7+(\infty)^3+1}{(\infty)^8-3(\infty)^6+5}\left(\dfrac{\infty}{\infty}\text{form}\right)$

$$\text{Now } \underset{x\to\infty}{\text{Lt}} \frac{x^7+x^3+1}{x^8-3x^6+5} = \underset{x\to\infty}{\text{Lt}} \frac{x^7\left(1+\frac{1}{x^4}+\frac{1}{x^7}\right)}{x^8\left(1-\frac{3}{x^2}+\frac{5}{x^8}\right)}$$

$$= \underset{x\to\infty}{\text{Lt}} \frac{1}{x}\cdot\frac{\left(1+\frac{1}{x^4}+\frac{1}{x^7}\right)}{\left(1-\frac{3}{x^2}+\frac{5}{x^8}\right)}$$

$$= \frac{1}{\infty}\cdot\frac{\left(1+\frac{1}{\infty}+\frac{1}{\infty}\right)}{\left(1-\frac{3}{\infty}+\frac{5}{\infty}\right)}$$

$$= 0.\frac{(1+0+1)}{(1-0+0)} = 0$$

EVALUATION OF EXPONENTIAL LIMITS

(*i*) $\underset{x\to 0}{\text{Lt}} \dfrac{2^x - 3^x}{x}$

Sol. (*i*) $\underset{x\to 0}{\text{Lt}} \dfrac{2^x - 3^x}{x}$ $\left(\dfrac{0}{0} \text{ form}\right)$

$$\underset{x\to 0}{\text{Lt}} \frac{(2^x - 1) - (3^x - 1)}{x} = \underset{x\to 0}{\text{Lt}} \left(\frac{2^x - 1}{x} - \frac{3^x - 1}{x}\right) = \log 2 - \log 3 = \log\left(\frac{2}{3}\right).$$

$$\underset{x\to 0}{\text{Lt}} \frac{a^x - 1}{x} = \log_e a.$$