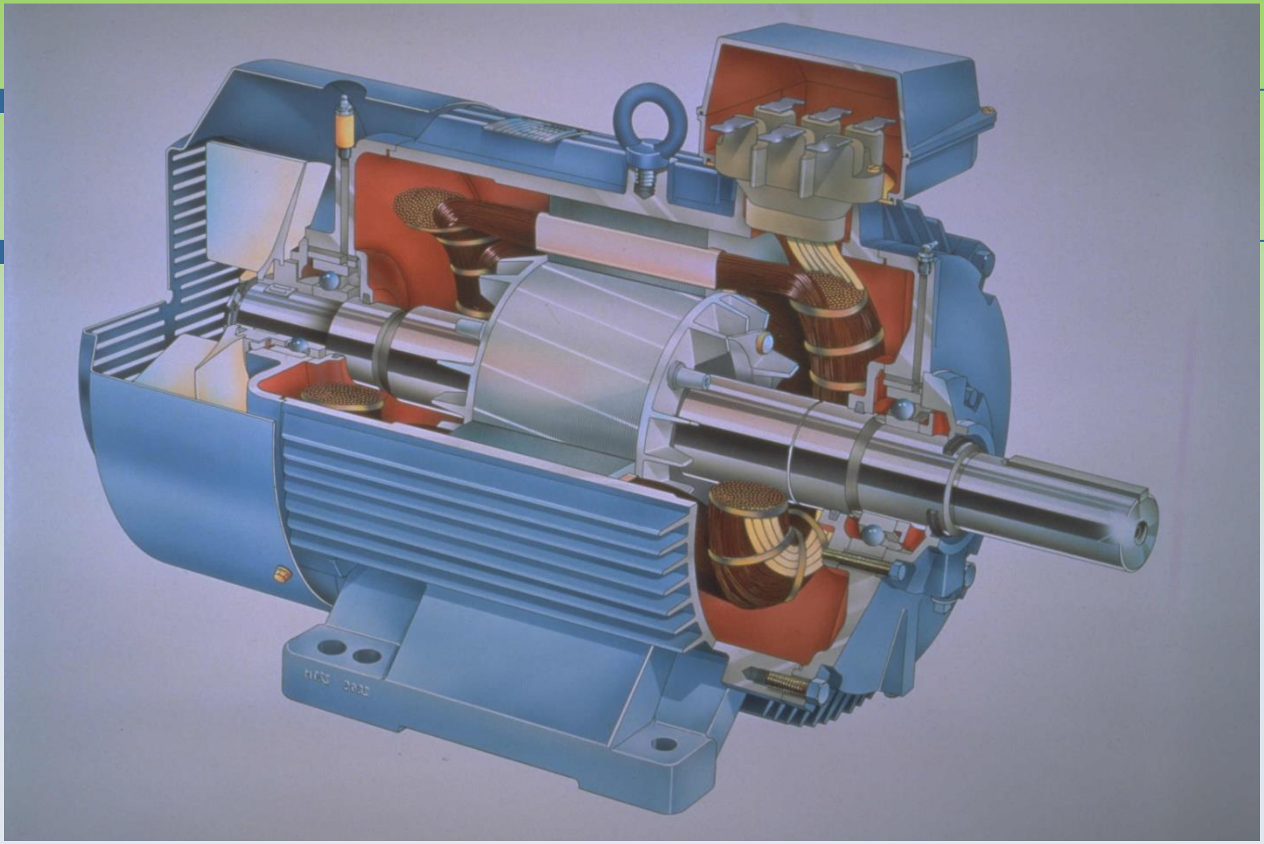
Induction Motors



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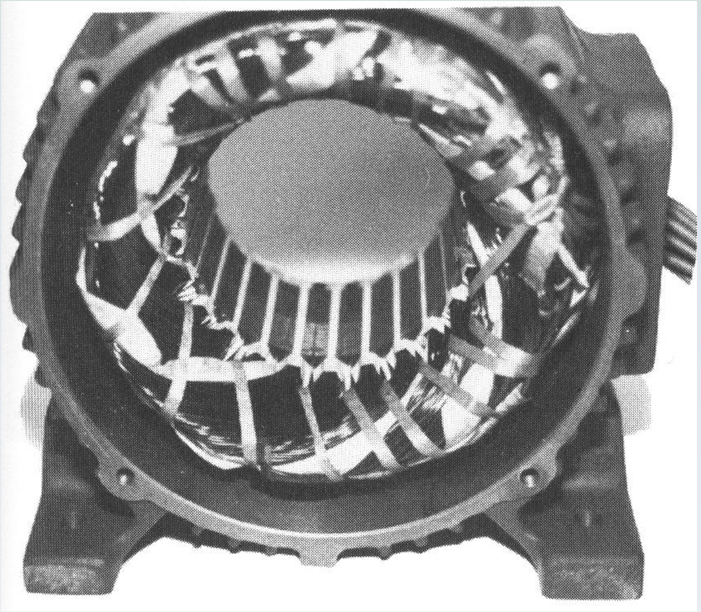
Introduction

- Three-phase induction motors are the most common and frequently encountered machines in industry
  - simple design, rugged, low-price, easy maintenance
  - wide range of power ratings: fractional horsepower to 10 MW
  - run essentially as constant speed from zero to full load
  - speed is power source frequency dependent
    - not easy to have variable speed control
    - requires a variable-frequency power-electronic drive for optimal speed control

Construction

- An induction motor has two main parts
  - a stationary stator
    - consisting of a steel frame that supports a hollow, cylindrical core
    - core, constructed from stacked laminations (why?), having a number of evenly spaced slots, providing the space for the stator winding



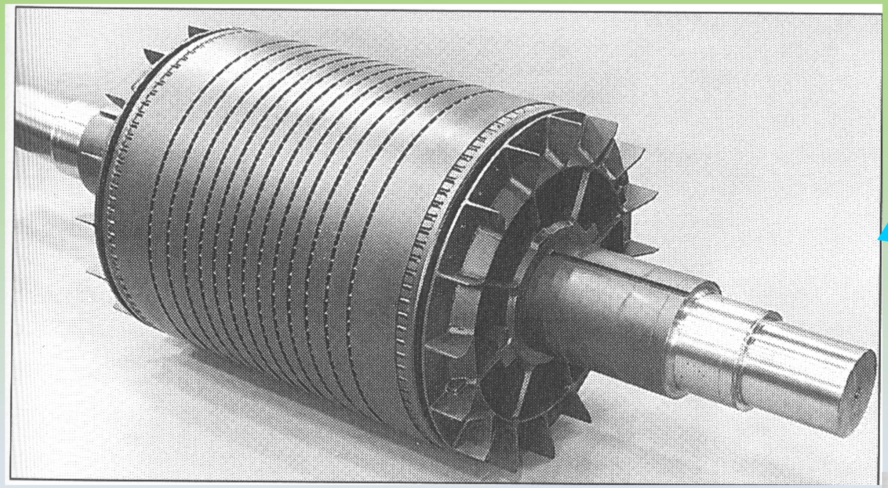
Stator of IM

Construction

- a revolving rotor
  - composed of punched laminations, stacked to create a series of rotor slots, providing space for the rotor winding
  - one of two types of rotor windings
  - conventional 3-phase windings made of insulated wire (wound-rotor) » similar to the winding on the stator
  - aluminum bus bars shorted together at the ends by two aluminum rings, forming a squirrel-cage shaped circuit (squirrel-cage)

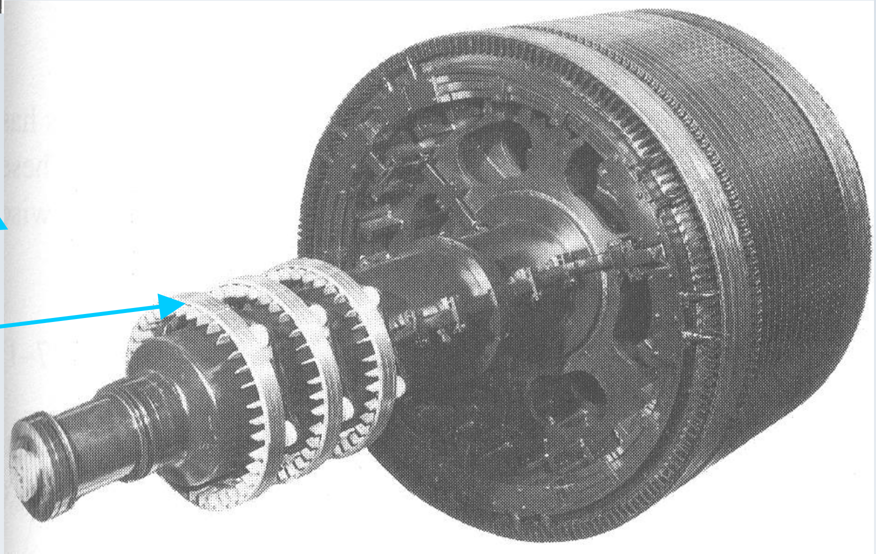
- Two basic design types depending on the rotor design
  - squirrel-cage
  - wound-rotor

Construction



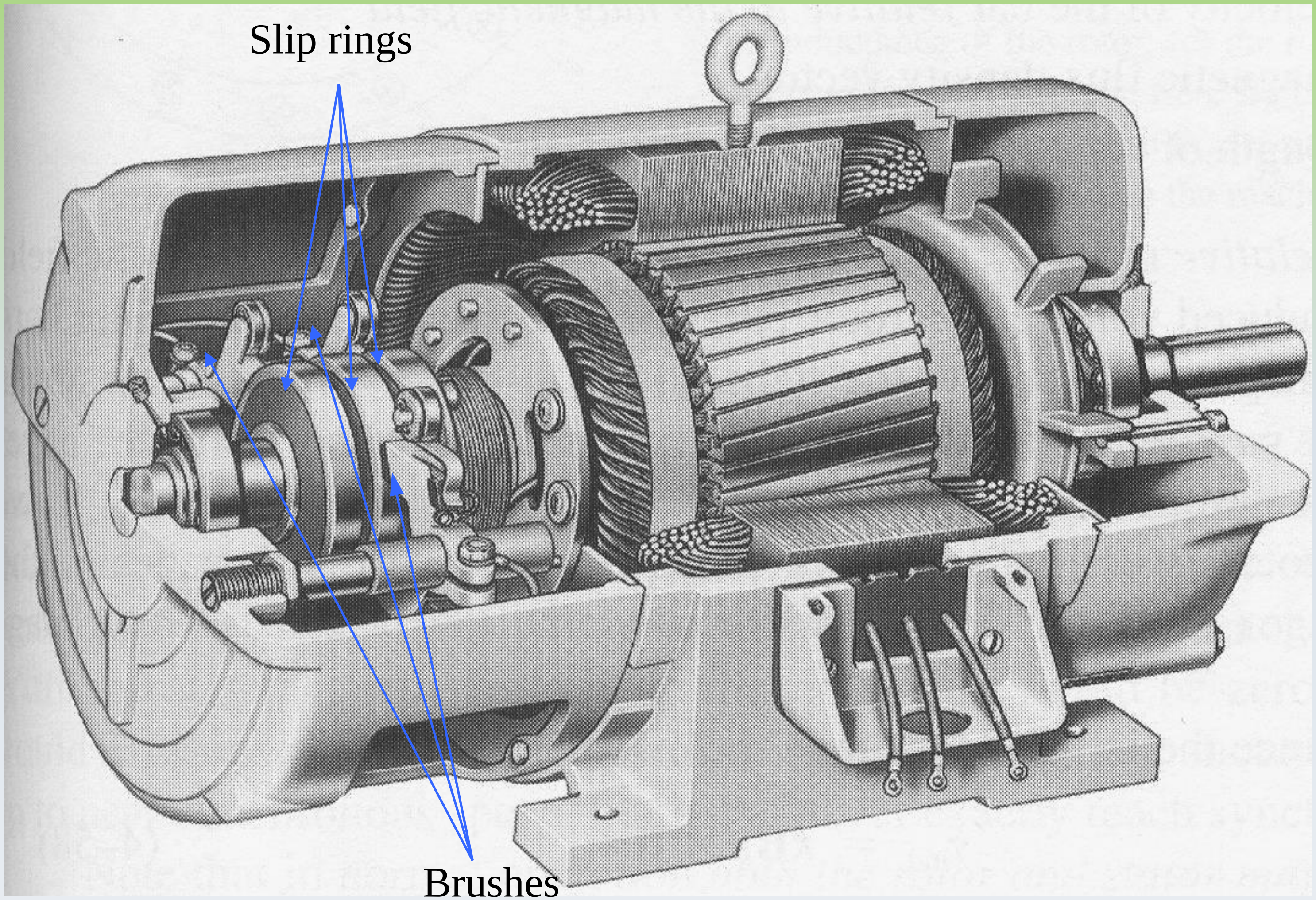
Squirrel cage rotor

Wound rotor



Notice the
slip rings

Construction

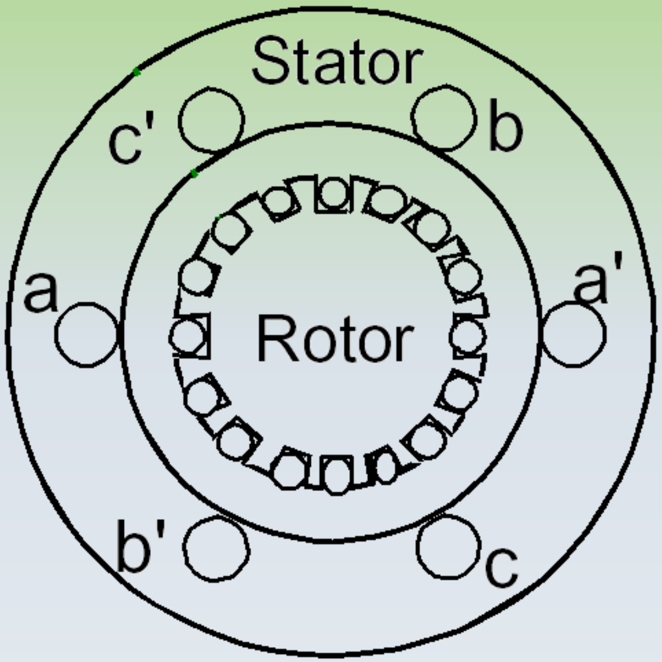


Cutaway in a typical wound-rotor IM. Notice the brushes and the slip rings

Rotating Magnetic Field

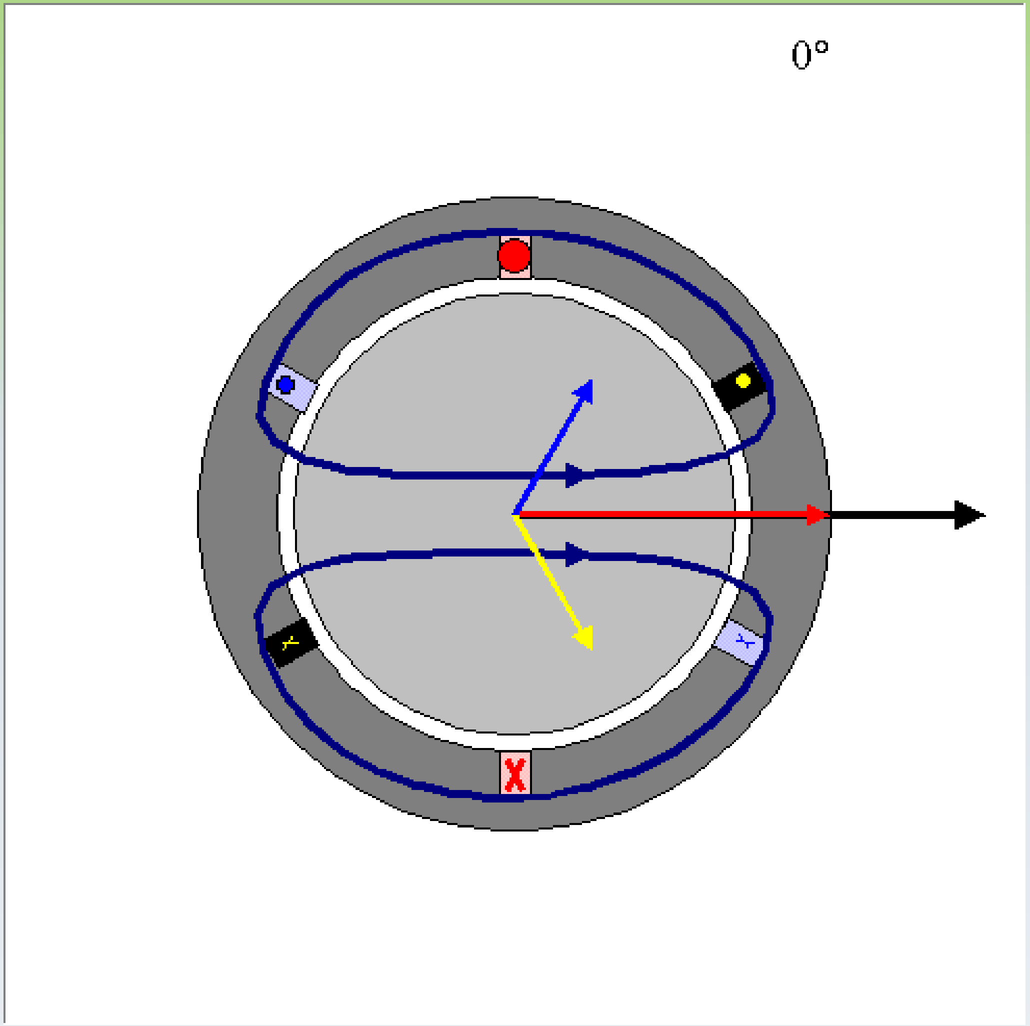
- Balanced three phase windings, i.e. mechanically displaced 120 degrees form each other, fed by balanced three phase source
- A rotating magnetic field with constant magnitude is produced, rotating with a speed

$$n_{sync} = \frac{120 f_e}{P} \quad rpm$$



Where f_e is the supply frequency and P is the no. of poles and n_{sync} is called the synchronous speed in *rpm* (revolutions per minute)

Rotating Magnetic Field



Principle of operation

- This rotating magnetic field cuts the rotor windings and produces an induced voltage in the rotor windings
- Due to the fact that the rotor windings are short circuited, for both squirrel cage and wound-rotor, and induced current flows in the rotor windings
- The rotor current produces another magnetic field
- A torque is produced as a result of the interaction of those two magnetic fields

$$\tau_{ind} = kB_R \times B_s$$

Where τ_{ind} is the induced torque and B_R and B_S are the magnetic flux densities of the rotor and the stator respectively

Induction motor speed

- At what speed will the IM run?
  - Can the IM run at the synchronous speed, why?
  - If rotor runs at the synchronous speed, which is the same speed of the rotating magnetic field, then the rotor will appear stationary to the rotating magnetic field and the rotating magnetic field will not cut the rotor. So, no induced current will flow in the rotor and no rotor magnetic flux will be produced so no torque is generated and the rotor speed will fall below the synchronous speed
  - When the speed falls, the rotating magnetic field will cut the rotor windings and a torque is produced

Induction motor speed

- So, the IM will always run at a speed lower than the synchronous speed
- The difference between the motor speed and the synchronous speed is called the *Slip*

$$n_{slip} = n_{sync} - n_m$$

Where n_{slip}= slip speed

n_{sync}= speed of the magnetic field

n_m = mechanical shaft speed of the motor

The Slip

$$s = \frac{n_{sync} - n_m}{n_{sync}}$$

Where s is the *slip*

Notice that : if the rotor runs at synchronous speed

$s = 0$

if the rotor is stationary

$s = 1$

Slip may be expressed as a percentage by multiplying the above eq. by 100, notice that the slip is a ratio and doesn't have units

Example 7-1 (pp.387-388)

A 208-V, 10hp, four pole, 60 Hz, Y-connected induction motor has a full-load slip of 5 percent

1. What is the synchronous speed of this motor?
2. What is the rotor speed of this motor at rated load?
3. What is the rotor frequency of this motor at rated load?
4. What is the shaft torque of this motor at rated load?

Solution

1. $$n_{sync} = \frac{120 f_e}{P} = \frac{120(60)}{4} = 1800\ rpm$$

2. $$n_m = (1-s)n_s$$
$$= (1-0.05)\times 1800 = 1710\ rpm$$

3. $$f_r = sf_e = 0.05\times 60 = 3Hz$$

4. $$\tau_{load} = \frac{P_{out}}{\omega_m} = \frac{P_{out}}{2\pi \frac{n_m}{60}}$$
$$= \frac{10\,hp\times 746\,watt/hp}{1710\times 2\pi\times(1/60)} = 41.7\ N.m$$

Problem 7-2 (p.468)

A 220-V, three-phase, two-pole, 50-Hz induction motor is running at a slip of 5 percent. Find:

(a) The speed of the magnetic fields in revolutions per minute

(b) The speed of the rotor in revolutions per minute

(c) The slip speed of the rotor

(d) The rotor frequency in hertz

SOLUTION

(a) The speed of the magnetic fields is

$$n_{sync} = \frac{120 f_e}{P} = \frac{120(50 \text{ Hz})}{2} = 3000 \text{ r/min}$$

(b) The speed of the rotor is

$$n_m = (1-s)\, n_{sync} = (1-0.05)(3000 \text{ r/min}) = 2850 \text{ r/min}$$

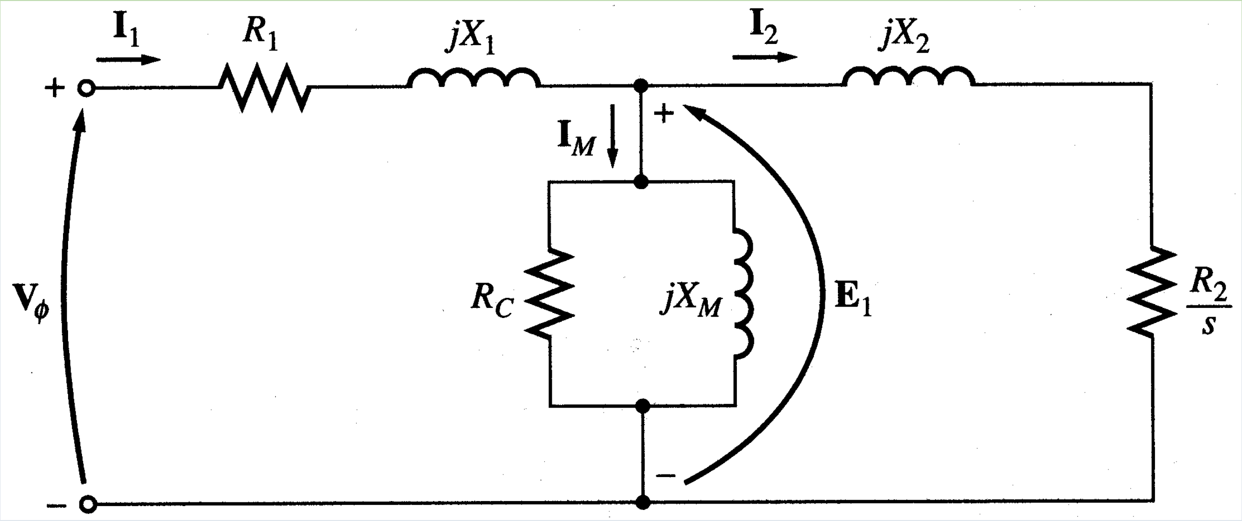
(c) The slip speed of the rotor is

$$n_{slip} = s n_{sync} = (0.05)(3000 \text{ r/min}) = 150 \text{ r/min}$$

(d) The rotor frequency is

$$f_r = \frac{n_{slip} P}{120} = \frac{(150 \text{ r/min})(2)}{120} = 2.5 \text{ Hz}$$

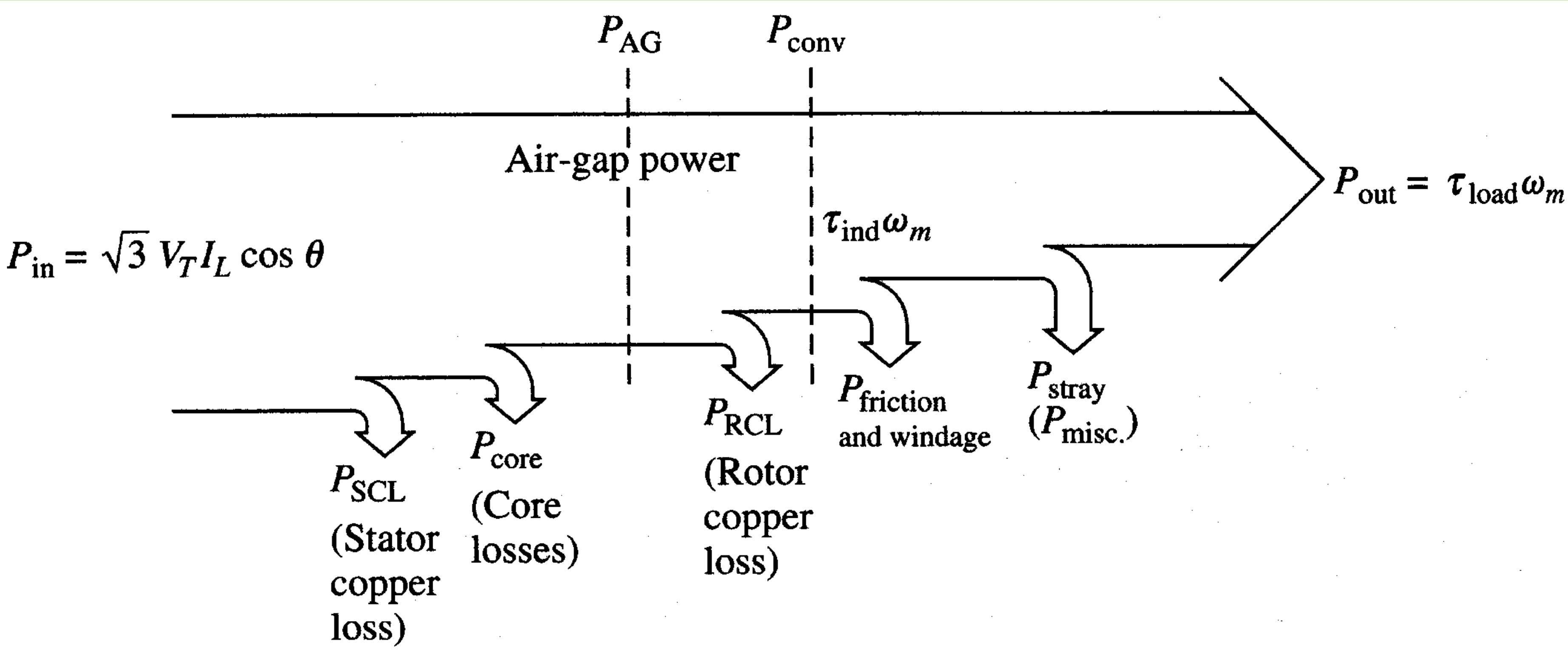
Equivalent Circuit



Power losses in Induction machines

- Copper losses
  - Copper loss in the stator (P_{SCL}) = $I_1^2 R_1$
  - Copper loss in the rotor (P_{RCL}) = $I_2^2 R_2$
- Core loss (P_{core})
- Mechanical power loss due to friction and windage
- How this power flow in the motor?

Power flow in induction motor



Power relations

$$P_{in} = \sqrt{3}\, V_L I_L \cos\theta = 3 V_{ph} I_{ph} \cos\theta$$

$$P_{SCL} = 3\, I_1^2 R_1$$

$$P_{AG} = P_{in} - (P_{SCL} + P_{core})$$

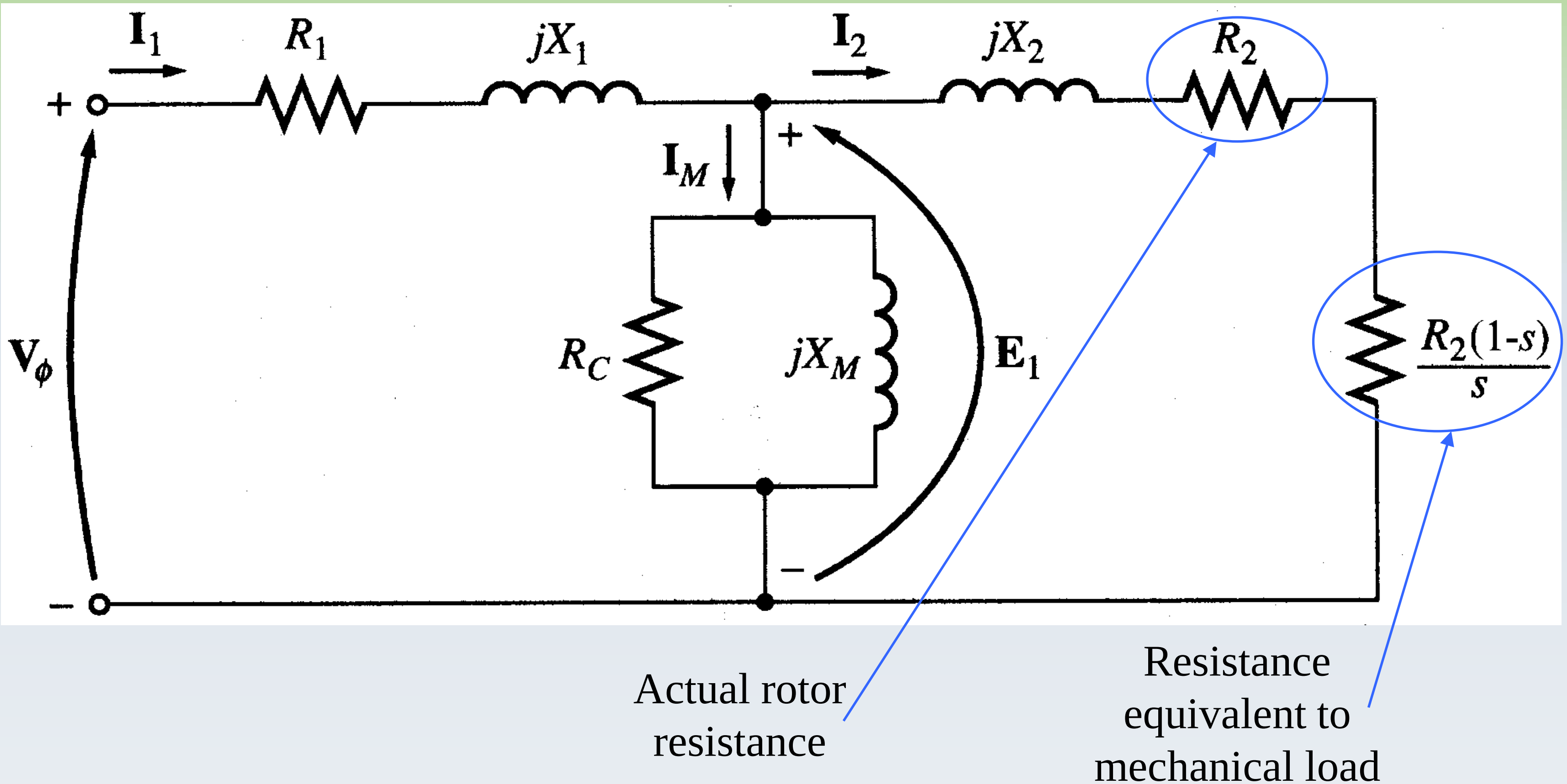
$$P_{RCL} = 3 I_2^2 R_2$$

$$P_{conv} = P_{AG} - P_{RCL}$$

$$P_{out} = P_{conv} - (P_{f+w} + P_{stray})$$

Equivalent Circuit

- We can rearrange the equivalent circuit as follows



Power relations

$$P_{in} = \sqrt{3}\, V_L I_L \cos\theta = 3\, V_{ph} I_{ph} \cos\theta$$

$$P_{SCL} = 3\, I_1^2 R_1$$

$$P_{AG} = P_{in} - (P_{SCL} + P_{core}) \quad = P_{conv} + P_{RCL} \quad = 3 I_2^2 \frac{R_2}{s} \quad = \frac{P_{RCL}}{s}$$

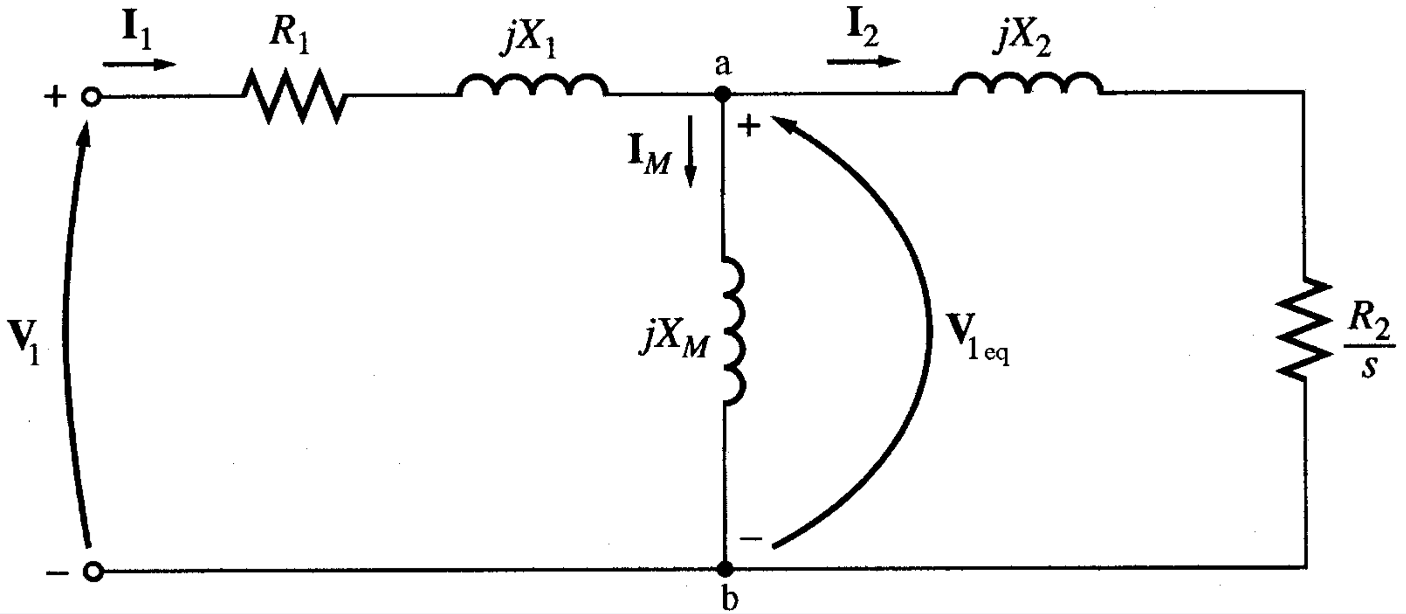
$$P_{RCL} = 3 I_2^2 R_2$$

$$P_{conv} = P_{AG} - P_{RCL} \quad = 3 I_2^2 \frac{R_2 (1-s)}{s} \quad = \frac{P_{RCL}(1-s)}{s}$$

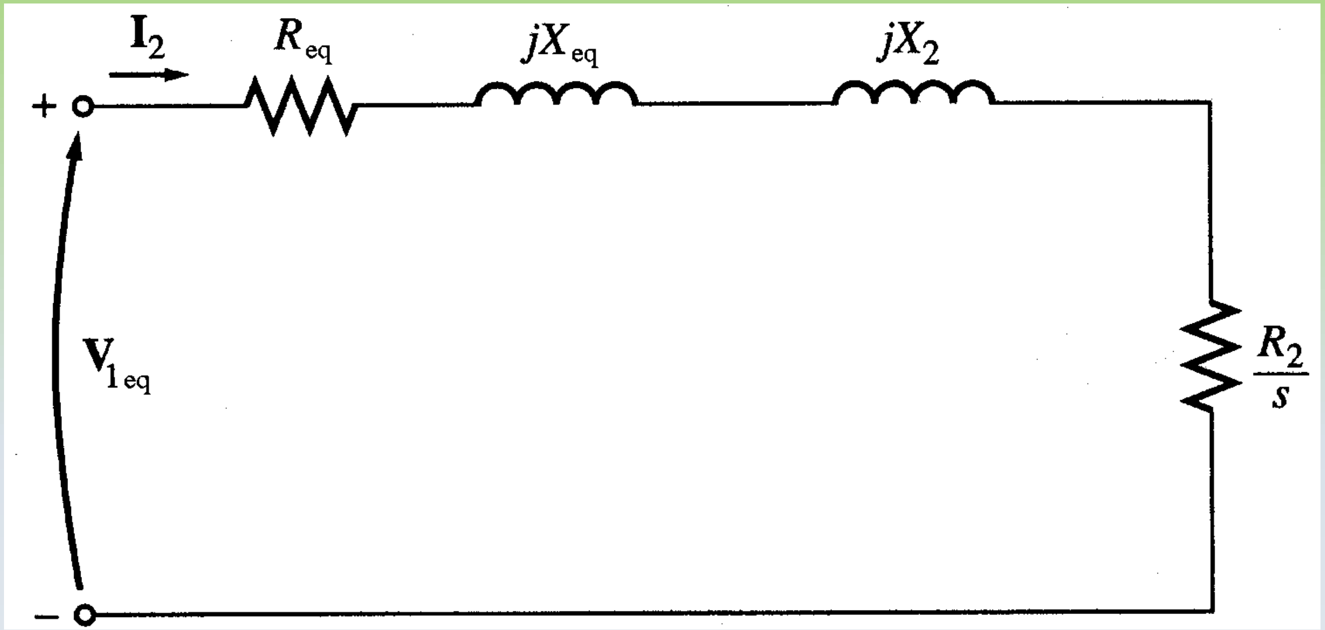
$$P_{out} = P_{conv} - (P_{f+w} + P_{stray})$$

Torque, power and Thevenin's Theorem

- Thevenin's theorem can be used to transform the network to the left of points 'a' and 'b' into an equivalent voltage source V_{1eq} in series with equivalent impedance $R_{eq}+jX_{eq}$



Torque, power and Thevenin's Theorem



$$V_{1eq} = V_1 \frac{jX_M}{R_1 + j(X_1 + X_M)}$$

$$R_{eq} + jX_{eq} = (R_1 + jX_1) // jX_M$$

Torque, power and Thevenin's Theorem

$$I_2 = \frac{V_{1eq}}{Z_T} = \frac{V_{1eq}}{\sqrt{\left(R_{eq} + \frac{R_2}{s}\right)^2 + (X_{eq} + X_2)^2}}$$

Then the power converted to mechanical (P_{conv})

$$P_{conv} = I_2^2 \frac{R_2(1-s)}{s}$$

And the internal mechanical torque (T_{conv})

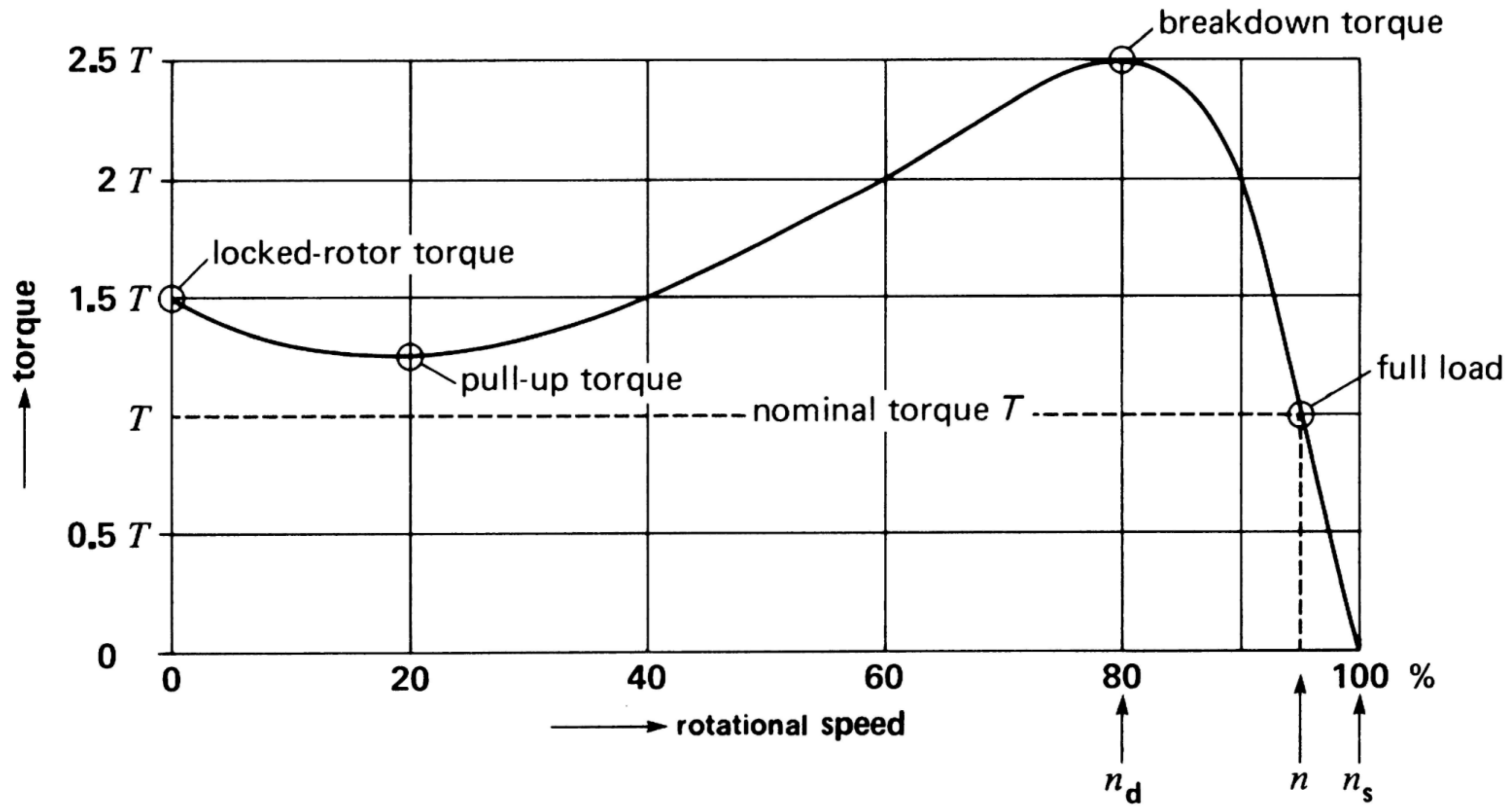
$$T_{conv} = \frac{P_{conv}}{\omega_m} = \frac{P_{conv}}{(1-s)\omega_s} = \frac{I_2^2 \frac{R_2}{s}}{\omega_s}$$

Torque, power and Thevenin's Theorem

$$T_{conv} = \frac{1}{\omega_s}\left(\frac{V_{1eq}}{\sqrt{\left(R_{eq}+\frac{R_2}{s}\right)^2+(X_{eq}+X_2)^2}}\right)^2\left(\frac{R_2}{s}\right)$$

$$T_{conv} = \frac{1}{\omega_s}\frac{V_{1eq}^2\left(\frac{R_2}{s}\right)}{\left(R_{eq}+\frac{R_2}{s}\right)^2+(X_{eq}+X_2)^2}$$

Torque-speed characteristics



Typical torque-speed characteristics of induction motor

Maximum torque

- Maximum torque occurs when the power transferred to R_2/s is maximum.
- This condition occurs when R_2/s equals the magnitude of the impedance $R_{eq} + j\,(X_{eq} + X_2)$

$$\frac{R_2}{s_{T_{\max}}} = \sqrt{R_{eq}^2 + (X_{eq} + X_2)^2}$$

$$s_{T_{\max}} = \frac{R_2}{\sqrt{R_{eq}^2 + (X_{eq} + X_2)^2}}$$

Maximum torque

- The corresponding maximum torque of an induction motor equals

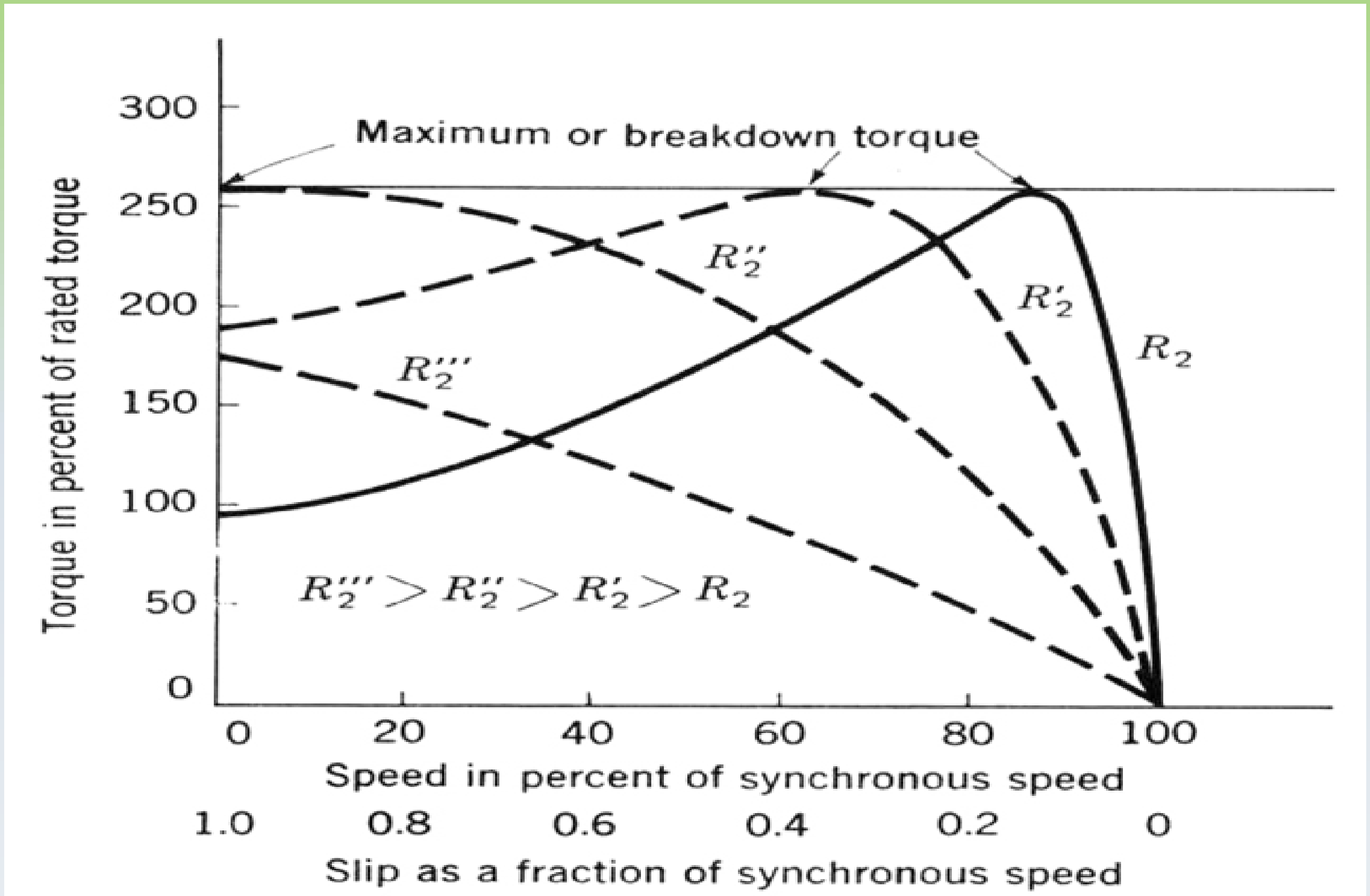
$$T_{\max} = \frac{1}{2\omega_s}\left(\frac{V_{eq}^2}{R_{eq} + \sqrt{R_{eq}^2 + (X_{eq} + X_2)^2}}\right)$$

- The slip at maximum torque is directly proportional to the rotor resistance R_2
- The maximum torque is independent of R_2

Maximum torque

- Rotor resistance can be increased by inserting external resistance in the rotor of a wound-rotor induction motor.
- The value of the maximum torque remains unaffected but the speed at which it occurs can be controlled.

Maximum torque



Effect of rotor resistance on torque-speed characteristic

Problem 7-5 (p.468)

A 50-kW, 440-V, 50-Hz, six-pole induction motor has a slip of 6 percent when operating at full-load conditions. At full-load conditions, the friction and windage losses are 300 W, and the core losses are 600 W. Find the following values for full-load conditions:

(a) The shaft speed n_m

(b) The output power in watts

(c) The load torque τ_{load} in newton-meters

(d) The induced torque τ_{ind} in newton-meters

(e) The rotor frequency in hertz

Solution to Problem 7-5 (p.468)

(a) The synchronous speed of this machine is

$$n_{\text{sync}} = \frac{120 f_e}{P} = \frac{120(50 \text{ Hz})}{6} = 1000 \text{ r/min}$$

Therefore, the shaft speed is

$$n_m = (1-s)\, n_{\text{sync}} = (1-0.06)(1000 \text{ r/min}) = 940 \text{ r/min}$$

(b) The output power in watts is 50 kW (stated in the problem).

(c) The load torque is

$$\tau_{\text{load}} = \frac{P_{\text{OUT}}}{\omega_m} = \frac{50 \text{ kW}}{(940 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)} = 508 \text{ N}\cdot\text{m}$$

(d) The induced torque can be found as follows:

$$P_{\text{conv}} = P_{\text{OUT}} + P_{\text{F\&W}} + P_{\text{core}} + P_{\text{misc}} = 50 \text{ kW} + 300 \text{ W} + 600 \text{ W} + 0 \text{ W} = 50.9 \text{ kW}$$

$$\tau_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_m} = \frac{50.9 \text{ kW}}{(940 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)} = 517 \text{ N}\cdot\text{m}$$

(e) The rotor frequency is

$$f_r = sf_e = (0.06)(50 \text{ Hz}) = 3.00 \text{ Hz}$$

Problem 7-7 (pp.468-469)

A 208-V, two-pole, 60-Hz Y-connected wound-rotor induction motor is rated at 15 hp. Its equivalent circuit components are

$R_1 = 0.200\ \Omega$ $R_2 = 0.120\ \Omega$ $X_M = 15.0\ \Omega$

$X_1 = 0.410\ \Omega$ $X_2 = 0.410\ \Omega$

$P_{mech} = 250\ \text{W}$ $P_{misc} \approx 0$ $P_{core} = 180\ \text{W}$

For a slip of 0.05, find

(a) The line current I_L

(b) The stator copper losses P_{SCL}

(c) The air-gap power P_{AG}

(d) The power converted from electrical to mechanical form P_{conv}

(e) The induced torque τ_{ind}

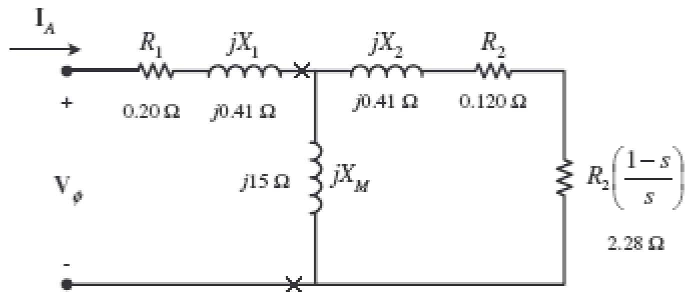
(f) The load torque τ_{load}

(g) The overall machine efficiency

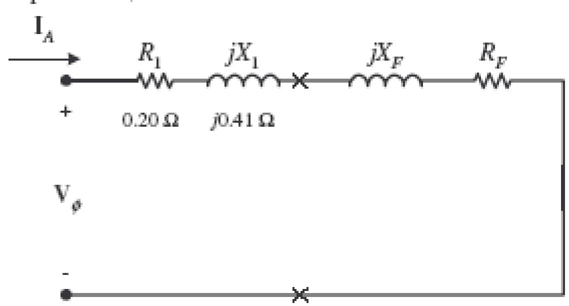
(h) The motor speed in revolutions per minute and radians per second

Solution to Problem 7-7 (pp.468-469)

SOLUTION The equivalent circuit of this induction motor is shown below:



(a) The easiest way to find the line current (or armature current) is to get the equivalent impedance Z_F of the rotor circuit in parallel with jX_M, and then calculate the current as the phase voltage divided by the sum of the series impedances, as shown below.



The equivalent impedance of the rotor circuit in parallel with jX_M is:

$$Z_F = \frac{1}{\frac{1}{jX_M}+\frac{1}{Z_2}} = \frac{1}{\frac{1}{j15\ \Omega}+\frac{1}{2.40+j0.41}} = 2.220 + j0.745 = 2.34\angle 18.5^\circ\ \Omega$$

The phase voltage is $208/\sqrt{3}$ = 120 V, so line current I_L is

Solution to Problem 7-7 (pp.468-469) – Cont'd

$$I_L = I_A = \frac{V_\phi}{R_1 + jX_1 + R_F + jX_F} = \frac{120\angle 0^\circ \text{ V}}{0.20\ \Omega + j0.41\ \Omega + 2.22\ \Omega + j0.745\ \Omega}$$

$$I_L = I_A = 44.8\angle -25.5^\circ \text{ A}$$

(b) The stator copper losses are

$$P_{\text{SCL}} = 3I_A^{\,2}R_1 = 3(44.8 \text{ A})^2(0.20\ \Omega) = 1205 \text{ W}$$

(c) The air gap power is $P_{AG} = 3I_2^{\,2}\frac{R_2}{s} = 3I_A^{\,2}R_F$

(Note that $3I_A^{\,2}R_F$ is equal to $3I_2^{\,2}\frac{R_2}{s}$, since the only resistance in the original rotor circuit was R_2/s, and the resistance in the Thevenin equivalent circuit is R_F. The power consumed by the Thevenin equivalent circuit must be the same as the power consumed by the original circuit.)

$$P_{AG} = 3I_2^{\,2}\frac{R_2}{s} = 3I_A^{\,2}R_F = 3(44.8 \text{ A})^2(2.220\ \Omega) = 13.4 \text{ kW}$$

(d) The power converted from electrical to mechanical form is

$$P_{\text{conv}} = (1-s)P_{AG} = (1-0.05)(13.4 \text{ kW}) = 12.73 \text{ kW}$$

(e) The induced torque in the motor is

$$\tau_{\text{ind}} = \frac{P_{AG}}{\omega_{\text{sync}}} = \frac{13.4 \text{ kW}}{(3600 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)} = 35.5 \text{ N}\cdot\text{m}$$

Solution to Problem 7-7 (pp.468-469) – Cont'd

(f) The output power of this motor is

$$P_{OUT} = P_{conv} - P_{mech} - P_{core} - P_{misc} = 12.73 \text{ kW} - 250 \text{ W} - 180 \text{ W} - 0 \text{ W} = 12.3 \text{ kW}$$

The output speed is

$$n_m = (1 - s)\, n_{sync} = (1 - 0.05)(3600 \text{ r/min}) = 3420 \text{ r/min}$$

Therefore the load torque is

$$\tau_{load} = \frac{P_{OUT}}{\omega_m} = \frac{12.3 \text{ kW}}{(3420 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)} = 34.3 \text{ N} \cdot \text{m}$$

(g) The overall efficiency is

$$\eta = \frac{P_{OUT}}{P_{IN}} \times 100\% = \frac{P_{OUT}}{3V_\phi I_A \cos\theta} \times 100\%$$

$$\eta = \frac{12.3 \text{ kW}}{3(120 \text{ V})(44.8 \text{ A})\cos 25.5^\circ} \times 100\% = 84.5\%$$

(h) The motor speed in revolutions per minute is 3420 r/min. The motor speed in radians per second is

$$\omega_m = (3420 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 358 \text{ rad/s}$$

Problem 7-19 (p.470)

A 460-V, four-pole, 50-hp, 60-Hz, Y-connected three-phase induction motor develops its full-load induced torque at 3.8 percent slip when operating at 60 Hz and 460 V. The per-phase circuit model impedances of the motor are

$R_1 = 0.33\ \Omega$ $\quad X_M = 30\ \Omega$

$X_1 = 0.42\ \Omega$ $\quad X_2 = 0.42\ \Omega$

Mechanical, core, and stray losses may be neglected in this problem.

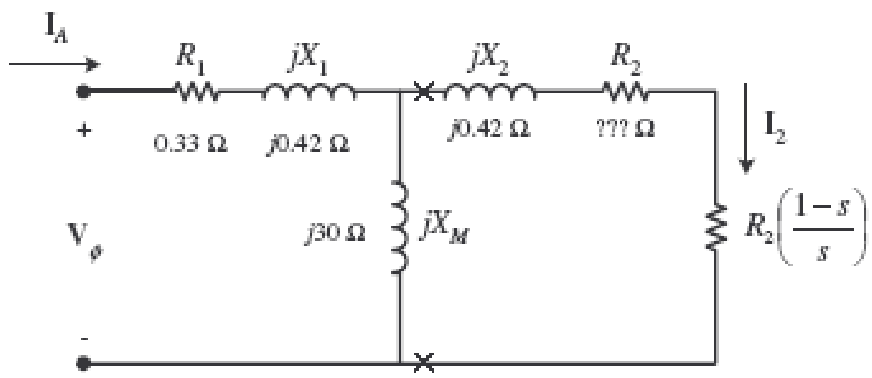
(a) Find the value of the rotor resistance R_2.

(b) Find τ_{max}, s_{max}, and the rotor speed at maximum torque for this motor.

(c) Find the starting torque of this motor.

Solution to Problem 7-19 (pp.470)

SOLUTION The equivalent circuit for this motor is



The Thevenin equivalent of the input circuit is:

$$Z_{TH} = \frac{jX_M(R_1 + jX_1)}{R_1 + j(X_1 + X_M)} = \frac{(j30\ \Omega)(0.33\ \Omega + j0.42\ \Omega)}{0.33\ \Omega + j(0.42\ \Omega + 30\ \Omega)} = 0.321 + j0.418\ \Omega = 0.527\angle 52.5°\ \Omega$$

$$\mathbf{V}_{TH} = \frac{jX_M}{R_1 + j(X_1 + X_M)}\mathbf{V}_\phi = \frac{(j30\ \Omega)}{0.33\ \Omega + j(0.42\ \Omega + 30\ \Omega)}(265.6\angle 0°\ \text{V}) = 262\angle 0.6°\ \text{V}$$

Solution to Problem 7-19 (pp.470) – Cont'd

(a) If losses are neglected, the induced torque in a motor is equal to its load torque. At full load, the output power of this motor is 50 hp and its slip is 3.8%, so the induced torque is

$$n_m = (1-0.038)(1800 \text{ r/min}) = 1732 \text{ r/min}$$

$$\tau_{\text{ind}} = \tau_{\text{load}} = \frac{(50 \text{ hp})(746 \text{ W/hp})}{(1732 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)} = 205.7 \text{ N}\cdot\text{m}$$

The induced torque is given by the equation

$$\tau_{\text{ind}} = \frac{3V_{\text{TH}}^2 R_2/s}{\omega_{\text{sync}}\left[(R_{\text{TH}} + R_2/s)^2 + (X_{\text{TH}} + X_2)^2\right]}$$

Substituting known values and solving for R_2/s yields

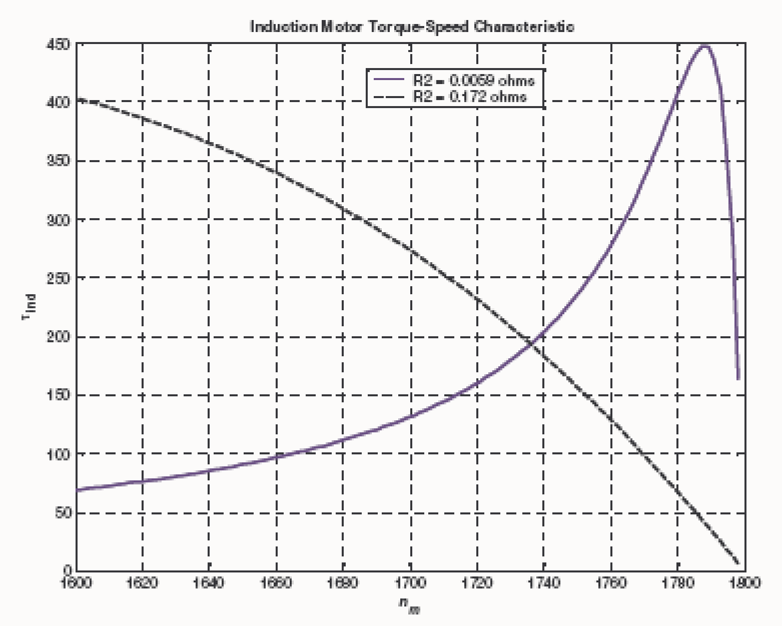
$$205.7 \text{ N}\cdot\text{m} = \frac{3(262 \text{ V})^2 R_2/s}{(188.5 \text{ rad/s})\left[(0.321 + R_2/s)^2 + (0.418 + 0.42)^2\right]}$$

$$38{,}774 = \frac{205{,}932\ R_2/s}{(0.321 + R_2/s)^2 + 0.702}$$

$$(0.321 + R_2/s)^2 + 0.702 = 5.311\ R_2/s$$

$$0.103 + 0.642 R_2/s + (R_2/s)^2 + 0.702 = 5.311\ R_2/s$$

Solution to Problem 7-19 (pp.470) – Cont'd



$$\left(\frac{R_2}{s}\right)^2 - 4.669\left(\frac{R_2}{s}\right) + 0.702 = 0$$

$$\frac{R_2}{s} = 0.156, \quad 4.513$$

$$R_2 = 0.0059\ \Omega,\ 0.172\ \Omega$$

These two solutions represent two situations in which the torque-speed curve would go through this specific torque-speed point. The two curves are plotted below. As you can see, only the 0.172 Ω solution is realistic, since the 0.0059 Ω solution passes through this torque-speed point at an unstable location on the back side of the torque-speed curve.

Solution to Problem 7-19 (pp.470) – Cont'd

(b) The slip at pullout torque can be found by calculating the Thevenin equivalent of the input circuit from the rotor back to the power supply, and then using that with the rotor circuit model. The Thevenin equivalent of the input circuit was calculate in part *(a)*. The slip at pullout torque is

$$s_{\max} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}}$$

$$s_{\max} = \frac{0.172\ \Omega}{\sqrt{(0.321\ \Omega)^2 + (0.418\ \Omega\ + 0.420\ \Omega)^2}} = 0.192$$

The rotor speed a maximum torque is

$$n_{\text{pullout}} = (1 - s)\ n_{\text{sync}} = (1 - 0.192)(1800\text{ r/min}) = 1454\text{ r/min}$$

and the pullout torque of the motor is

$$\tau_{\max} = \frac{3V_{TH}^2}{2\omega_{\text{sync}}\ \left[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}\right]}$$

$$\tau_{\max} = \frac{3(262\text{ V})^2}{2(188.5\text{ rad/s})\ \left[0.321\ \Omega + \sqrt{(0.321\ \Omega)^2 + (0.418\ \Omega\ + 0.420\ \Omega)^2}\right]}$$

$$\tau_{\max} = 448\text{ N}\cdot\text{m}$$

(c) The starting torque of this motor is the torque at slip $s = 1$. It is

$$\tau_{\text{ind}} = \frac{3V_{TH}^2 R_2 / s}{\omega_{\text{sync}}\ \left[(R_{TH} + R_2 / s)^2 + (X_{TH} + X_2)^2\right]}$$

$$\tau_{\text{ind}} = \frac{3(262\text{ V})^2 (0.172\ \Omega)}{(188.5\text{ rad/s})\ \left[(0.321 + 0.172\ \Omega)^2 + (0.418 + 0.420)^2\right]} = 199\text{ N}\cdot\text{m}$$